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Think it, don't overthink it: Learning from a children's puzzle

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Abstract - In this article, we provide a simple example of a two-player game with asymmetrical resource to show how strategic choice of bounded rationality can be helpful for the less endowed contestant. We also demonstrate through the game that there are situations where being ‘primitively rational’ is a more efficient choice than being (or trying to be) hyper-rational. It underscores a peculiar situation where choosing to think less proves to be a superior strategy to overthinking. We use a lender-borrower game and a children’s puzzle to demonstrate our argument.

1. Preamble

Let us think of a situation between a borrower and a lender. The lender has a upper limit of money available to her for lending, and she cannot provide more than that. If she has to provide more than that, she goes into debt herself. On the other hand, if the borrower is borrowing for the first time, a debt puts them in a precarious position for they have to return the money back at some point. However, if the borrower already has a debt, a larger debt will temporarily provide a relief to the borrower. So, for a borrower, a larger debt than the previous one gives the borrower some relief, whereas for lender, a smaller borrowing (means a larger savings) than the maximum amount available puts the lender at ease. Neither of these two parties know the private information that is available only to themselves. For a borrower, the previous debt history (and thus the capacity to pay back) is a private information, whereas the capacity to lend money is a private information for the lender. On another level, the borrower might have undisclosed assets that secretly adds to her payback capacity, and the lender might have a private insurance scheme to save her from disasters. The question this paper tries to address is, is there any situation where the borrower, who is clearly the economically weaker one, trumps the lender and puts the lender in a precarious position while temporarily remaining safe themselves?

We are looking at a specific situation in this paper. For the analysis of this paper, if a lender and borrower meet, they must agree to lend and borrow. A real life situation of indiscriminate approvals of debt follows this rule. That is precisely what happened before the housing bubble burst in the USA. So in this game, each player has a history that is personal and private to them, and they do not understand the consequence of the transaction before the transaction happens. It means, the borrower has to take any money that the lender gives, and the lender has to accept the borrower's payback capacity without question. Of course, the lender can come with a financial backup, and the borrower can also do so in order to save themselves from an impending doom.

This situation conspicuously resonates with a children's puzzle. Consider the following puzzle:

1.1. **Dragon and Poison.** We quote the following puzzle by Tanton [2019].

> You and a dragon have agreed to take part in the following "game". (I am not sure why but, well, that is how it is). At noon today you will each bring to the local coffee house a goblet of poison. The dragon will take a sip of poison from your goblet and then a sip from his own. You will take a sip of poison from the dragon's goblet and then a sip from your own. You will then each sit and wait for the results.
>
> Let me tell you about the poison.
>
> There is only one type of poison available to each of you and it comes in varying strengths of potency. A single sip of any potency is enough for quite a detrimental effect (namely your or the dragon's complete demise), but it will take a few hours to act. There is an

antidote to the poison: a sip of a stronger dose of the poison. Taking two sips - one dose followed by a stronger dose - has the same effect as not taking any poison at all. However, taking a second dose of equal or weaker potency will not help your predicament one whit.

Let me tell you something about the dragon.

She has access to the most potent strength of poison of all. (And you don't!)

So here is the challenge.

Given this knowledge of how poison works and the fact that the dragon might bring the most potent sample of all, is there a means for you to survive this cheery game?

Psst: You can drink something else before coming to the duel as well. The other party does not have to know that.

2. Game Model

To model the puzzle as a game, we need to have two players, the Dragon (D) and the Knight (K) [or, if you remember, the lender and the borrower]. Let $S = \{0, 1, \dots, h\}$ denote the possible strengths of the potency of the poison available. D can choose any $s_D \in S$, whereas K can choose any $s_K \in S^- = \{0, 1, \dots, h-1\}$. The restriction of the choice set for the Knight resonates to the fact that a borrower is never as much financially strong as the lender. The choice for each player is a couple (s_{D_1}, s_{D_2}) and (s_{K_1}, s_{K_2}) where index 1 indicates the poison to be taken before the duel and index 2 indicates the poison the player brings to the duel. The poison in the real life case is money. It means that the borrower offers a guarantee of payback capacity s_{K_2}, while the lender offers a guaranteed sum of money s_{D_2}. After putting these on the table, they get to know about each others choice. Moreover, they might or might not have secured some guarantee elsewhere, without knowledge of the other party. For the borrower, it may be some inheritance s_{K_1} whereas for the lender it may be some insurance amount s_{D_1}.

In this case, the payoff function of the Dragon looks like:

$u_D(s_{D_1}, s_{D_2}, s_{K_2}) = 1$ if

(1) either all $s = 0$ or
(2) if $s_{D_1} = 0$ and $s_{K_2} > 0$ and $s_{D_2} > s_{K_2}$ or
(3) if $s_{D_1} > 0$ and $s_{K_2} = 0$ and $s_{D_2} > s_{D_1}$ or
(4) if $s_{D_1} > s_{K_2} > 0$ and $s_{D_2} > s_{D_1}$ or
(5) if $s_{K_2} > s_{D_1} > 0$ and $s_{D_2} > s_{K_2}$

$u_D = 0$ for all other cases

This shows that for every choice of s_{K_2}, the best response of the Dragon is to bring the poison of highest potential, regardless of what the dragon drinks before the duel. Considering that the Knight knows this, the best choice for the knight is to keep $s_{K_1} = 1$, in order to survive. Also, the knight will know that he cannot kill the dragon if the dragon's strategy is $(s_{D_1} = 1, s_{D_2} = h)$. So his best response will be $(s_{K_1} = 1, s_{K_2} = 0)$. Against this strategy of the Knight, Dragon's best response is ($s_{D_1} = 1$, $s_{D_2} = h$). Hence, this is the **Nash Equilibrium**. At the outcome of this Nash Equilibrium, both Dragon and Knight survives the game. In order to reach the equilibrium, both the Dragon and the Knight has to be hyper-rational. But what if they are not? What if they are human beings with emotions and ego and whatnot? We delve a bit deeper in the following part of the paper.

2.1. **Changing utility structure.** Now, if we change the structure of the utility from only considering 'being alive' by including 'saving my self-respect' into consideration, we can say that for each player, the preference structure of the payoffs (Dragon, Knight) is as follows, where 0 means death and 1 means being alive. We have assumed that 'self-respect' is better saved when both die compared to when one dies alone.

$$\text{For Dragon: } (1,0) \succ (1,1) \succ (0,0) \succ (0,1)$$

$$\text{For Knight: } (0,1) \prec (1,1) \prec (0,0) \prec (1,0)$$

This transformation of the utility can be achieved by the following transformation:

$$\pi_i = \frac{a^{u_i}}{b^{u_j}}, \quad a > b, \quad a, b > 1$$

Considering $a = 3$ and $b = 2$ arbitrarily and without loss of generality,

$$\pi_D = \frac{3^{u_D}}{2^{u_K}}$$
$$\pi_K = \frac{3^{u_K}}{2^{u_D}}$$

Payoff/Outcome	1,0	1,1	0,0	0,1
(π_D, π_K)	3, 0.5	1.5, 1.5	1,1	0.5, 3

Table 1. Utility associated with payoff

As the preference towards poison, for their antidote property, shows a weak monotonicity, we can reduce the strategy space to essentially 3 choices for each player, i.e. No poison (0), Lowest poison (1), and Highest poison (h for Dragon and say $m = h - 1$ for Knight). With this reduced strategy space, we can configure the payoff matrix as shown in Table 2, with row player as Dragon and column player as Knight.

Table 2 shows that the after the change in the utility realization, there is no Pure Strategy Nash Equilibrium. The game oscillates between the 4 cells marked in red. Question is, where does the rationalizing stop?

3. Analysing through the Lens of Bounded Rationality: Level-K Thinking

Level-k theory is a competing theory to Cognitive Hierarchy Theory [Stahl, 1993] but is similar to Cognitive Hierarchy Theory in the sense that player types are drawn from a hierarchy of levels of iterated rationalizability. The hierarchy begins with some very naive type. This completely non-strategic "level-zero" player will choose actions without regard to the actions of other players. Such a player is said to have zero-order beliefs. A one level higher sophisticated type believe the population consists of all naive types. This slightly more sophisticated (the level one) player

	0,1	**0,m**	**1,0**	**1,m**	**m,0**	**m,1**	**0,0**	**1,1**	**m,m**
0,1	1,1	1,1	1,1	0.5, 3	1,1	1,1	1,1	1,1	1,1
0,h	1,1	1,1	0.5, 3	3,0.5	0.5, 3	3,0.5	1,1	3,0.5	3,0.5
1,0	1,1	3,0.5	1,1	1.5, 1.5	0,0	1,1	0.5, 3	1,1	3,0.5
1,h	3,0.5	3,0.5	1.5, 1.5	3,0.5	3,0.5	3,0.5	3,0.5	3,0.5	3,0.5
h,0	1,1	1,1	1,1	0.5, 3	1,1	1,1	0.5, 3	1,1	1,1
h,1	1,1	1,1	1,1	0.5, 3	1,1	1,1	0,0	1,1	1,1
0,0	1,1	1,1	3,0.5	0.5, 3	3,0.5	1,1	1.5, 1.5	1,1	1,1
1,1	1,1	1,1	1,1	0.5, 3	1,1	1,1	1,1	1,1	1,1
h,h	1,1	1,1	0.5, 3	1,1	0.5, 3	1,1	1,1	1,1	1,1

Table 2. Payoff matrix for the modified utility structure

believes that the other players will act non-strategically; his or her action will be the best response consistent with those first-order beliefs. The next level believes the population consists of the first level. This more sophisticated (level two) player acts on the belief that the other players are level one. This pattern continues for higher-level players, but each player has only a finite depth of reasoning, meaning that individual players have a limit to the depth to which they can reason strategically. Level-k theory assumes that players in strategic games base their decisions on their predictions about the likely actions of other players. According to level-k, players in strategic games can be categorized by the "depth" of their strategic thought. It is thus heavily focused on bounded rationality. In its basic form, level-k theory implies that each player believes that he or she is the most sophisticated person in the game. Players at some level k will neglect the fact that other players could also be level-k, or even higher. This has been attributed to many factors, such as "maintenance costs" or simply overconfidence. Level-k models were introduced to describe experimental data by Stahl and Wilson [1995] and Nagel [1995], and were further studied experimentally by Ho et al. [1998], Costa-Gomes et al. [2001], Costa-Gomes and Weizsäcker [2008], and Costa-Gomes and Crawford [2006] among others.

3.1. **Defining Actions.** In this section, we define the actions each player will take based on the level they think they are. The Level-0 action is defined as the most naive action to kill the opponent, that is, to bring the strongest poison possible, and the actions for the subsequent levels are defined as the best responses as per the Level-k theory. For example, the Level-1 action for the Dragon will be the best response against Level-0 action of the Knight, and Level-3 action for the Knight will be the best response to Level-2 action of the Dragon.

For Dragon:

- Level-0: Drink water, Bring the strongest poison
- Level-1: Drink the weakest poison, bring water
- Level-2: Drink water, bring water
- Level-3: Drink weakest, bring strongest
- Level-4: Level-1

For Knight:

- Level-0: Drink water, Bring the strongest poison
- Level-1: Drink the weakest poison, bring water

- Level-2: Drink water, bring water
- Level-3: Drink weakest, bring strongest
- Level-4: Level-1

3.2. **Level-k Utility Structure.** Let us assume that the Dragon is a Level-i player and the Knight is a Level-j player.

Outcome of the game for matching levels for the players are as follows:

- 0,0: Dragon lives
- 1,1: Both die
- 2,2: Both live
- 3,3: Both die

Outcome of the game for non-matching levels (i, j) are as follows:

- $i < j$:
  - 0,1: Knight lives. Payoff (0,1). Utility (0.5, 3)
  - 0,2: Both die. Payoff (0,0). Utility (1,1)
  - 0,3: Dragon lives. Payoff (1,0). Utility (3, 0.5)
  - 1,2: Knight lives. Payoff (0,1). Utility (0.5, 3)
  - 1,3: Both live. Payoff (1,1). Utility (1.5, 1.5)
  - 2,3: Knight lives. Payoff (0,1). Utility (0.5, 3)
- $j < i$:
  - 1,0: Both live. Payoff (1,1). Utility (1.5, 1.5)
  - 2,0: Both die. Payoff (0,0). Utility (1,1)
  - 3,0: Both die. Payoff (0,0). Utility (1,1)
  - 2,1: Dragon lives. Payoff (1,0). Utility (3, 0.5)
  - 3,1: Both live. Payoff (1,1). Utility (1.5, 1.5)
  - 3,2: Dragon lives. Payoff (1,0). Utility (3, 0.5)

3.3. **Payoff Calculation.** Now, let us assume that the Dragon believes that the Knight is a Level-k player with probability p_k, $k = 0, 1, \ldots, i-1$, and Knight believes that the Dragon is a Level-d player with probability q_d, $d = 0, 1, \ldots, j-1$.

Hence, Dragon's expected utility when it is at level:

- 0: 3
- 1: 1.5
- 2: $p_0 + 3p_1$
- 3: $p_0 + 1.5p_1 + 3p_2$

Similarly, Knight's expected utility when it is at level:

- 0: 0.5
- 1: 3
- 2: $q_0 + 3q_1$
- 3: $0.5q_0 + 1.5q_1 + 3q_2$

So, for Dragon, the weakly dominating choice is to be at Level-0 while for Knight the weakly dominating choice is to be at Level-1. The outcome is {(0,h), (1,0)} in terms of the converging strategy, and the Knight lives while the Dragon dies.

4. Concluding Remarks

It is very interesting to note that according to the logic of Cognitive Hierarchy, the weaker contender wins the game if she can strategically choose her intelligence level. Another remarkable feature that this game brings out is that although people have choice to be infinitely intelligent, the dominating choice does not go beyond Level-1. Which means, for all practical purposes, hyper-rational strategic investment may not be the best choice always, and not only that, being strategic at the minimum level might get the job done. In the game presented in this paper, hyper-rationality was not able to reach an equilibrium point, whereas bounded rationality assumptions showed the path.

Also it means that if you have to win the game with fewer resources, you must be one level ahead in rational thinking. And this is what happened in the housing bubble in the USA during the financial crisis. The lenders did not have any insurance and they brought their whole money in the market for disbursal, whereas the borrower was ready to bluff, with a private information of secured inheritance or some other kind of guarantee. And we all know how the story ends with a victorious Knight and a slayed Dragon.

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